Matrices - Determinants

1) If $a+b+c=0$ and $\begin{vmatrix} a-x & a & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix}=0$ then prove that either x = 0 or $x=\pm\sqrt{\frac{3}{2}}(a^2+b^2+c^2)$

2) If $A=\begin{bmatrix} p & q \\ r & -p \end{bmatrix}$ is such that $A^2=i$ then find the value of $1-P^2+qr$

3) If $A=\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $A+A^T=I$. Find the possible values of θ $\theta=\Pi/3$

4) Inverse of a square matrix is unique. Give an example to prove it?

5) Prove that $\begin{vmatrix} x-3 & x-4 & x-a \\ x-2 & x-3 & x-b \\ x-1 & x-2 & x-c \end{vmatrix}=0$, where a, b, c are in A.P.

6) Using properties of Determinants prove that : $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+ac & c^2 \end{vmatrix}=4a^2b^2c^2$

7) Express $\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$ as sum of the symmetric and skew symmetric matrices.

8) Prove that $\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ca & bc & (a+b)^2 \end{vmatrix}=2abc(a+b+c)^2$ (Use properties to prove the above)

9) Prove the determinant $\begin{vmatrix} x & -\sin\alpha & \cos\alpha \\ \sin\alpha & -x & 1 \\ \cos\alpha & 1 & x \end{vmatrix}$ is independent as α (Ans: Scalar term)

10) The total number of elements in a matrix represents a prime number. How many possible orders a matrix can have. 2

11) Find the matrix X such that : $\begin{bmatrix} 2 & -1 \\ 0 & 1 \\ -2 & 4 \end{bmatrix}X=\begin{bmatrix} -1 & -8 & -10 \\ 3 & 4 & 0 \\ 10 & 20 & 10 \end{bmatrix}$

12) If $f(x) = 3x^2-9x+7$, then for a square matrix A, write f(A) $(3A^2-9A+7I)$

13) Prove that $\begin{bmatrix} (a+1) & (a+2) & (a+2) & 1 \\ (a+2) & (a+3) & a+3) & 1 \\ (a+3) & (a+4) & (a+4) & 1 \end{bmatrix}=-2$

14) If $\begin{bmatrix} Cos^2A & CosASinA \\ CosASinA & Sin^2A \end{bmatrix}$, $Y = \begin{bmatrix} Cos^2B & CosBSinB \\ CosBSinB & Sin^2B \end{bmatrix}$

then show that XY is a zero matrix, provided (A-B) is an odd multiple of $\Pi/2$

15) Give that x = -9 is a root of $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$ find the other roots. Hint: Evaluate, find other roots.

16) If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ find x and y such that $A^2 + xI = yA$. Find A^{-1}

17) If P and Q are equal matrices of same order such that PQ = QP, then prove by induction that $PQ^n = Q^nP$. Further, show that $(PQ)^n = P^n \times Q^n$

18) If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, then $A^5 = ?$ $\qquad I_3$

19) If A and B are two matrices such that AB = B and BA = A then A^2+B^2 = ____?____\

20) If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ then prove that $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$ $n \varepsilon N$

21) Find the values of a, b, c if the matrix

$A = \begin{vmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{vmatrix}$ satisfy the equation $A^I A = I_3$

22) Assume X, Y, Z, W and P are matrices of order (2 x n), (3xk), (n x 3) and (p x k) respectively, then the restriction on n, k and p so that py + my will be defined are : (k=3, p=n)

23) Let A and B be 3 x 3 matrices such that $A^T = -A$ and $B^T = B$. Then the matrix λ (AB+3BA) is skew symmetric matrix for λ. $\qquad (\lambda = 3)$

24) If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$, use the result to find A^4 $\qquad \begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}$

25) For what value of 'K' the matrix $\begin{bmatrix} k & 2 \\ 3 & 4 \end{bmatrix}$ has no inverse. $\qquad$ (K = 3/2)

26) If A is a non-singular matrix of order 3 and |A| = -4 Find |adj A| $\qquad$ (16)

27) Given $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ such that $|A| = -10$. Find $a_{11}c_{11} + a_{12}c_{12}$ $\qquad$ (10)

28) If $\begin{vmatrix} x & b & c \\ a & y & c \\ a & b & z \end{vmatrix} = 0$, then find the value of $\frac{x}{x-a} + \frac{y}{y-b} + \frac{z}{z-c}$ $\qquad$ (2)

29) If A = $\begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$ satisfies the equation :

x^2-6x+17 = 0 find A^{-1} Ans: $\left(\frac{1}{17}\begin{bmatrix} 4 & 3 \\ -3 & 2 \end{bmatrix}\right)$

30) Find the matrix x if $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} x \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 3 & -2 \end{bmatrix}$

$X = -\frac{1}{4}\begin{bmatrix} -53 & 18 \\ 25 & -10 \end{bmatrix}$

31) If P(a) = $\begin{bmatrix} \cos a & -\sin a & 0 \\ \sin a & \cos a & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then show that $[P(x)]^{-1} = [P(-x)]$

32) If two matrices A^{-1} and B are given how to find $(AB)^{-1}$ verify with an example.

(Find B^{-1} then find $B^{-1} \times A^{-1}$)

33) If A = $\begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$ Verify $(AdjA)^{-1} = adj(A^{-1})$

34) Find the values of a and b such that $A^2 + aI = bA$ where A = $\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ (a=b=8)

35) If $P(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ show that $p(\alpha) \times p(\beta) = p(\alpha+\beta)$

36) If A = $\begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$ then prove by Mathematical Induction that : $A^n = \begin{bmatrix} \cos n\theta & i\sin n\theta \\ i\sin n\theta & \cos n\theta \end{bmatrix}$

37) If A = $\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$ find Matrix B such that AB = I, Ans : $B = \begin{bmatrix} 1 & 2 \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$

38) If x, y, z are positive and unequal show that the value of determinant $\begin{vmatrix} x & y & z \\ y & z & x \\ z & x & y \end{vmatrix}$ is negative.

39) if $A+B+C=\Pi$, show that $\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix} = 0$

40) Find the quadratic function defined by the equation $f(x) = ax^2+bx+c$ if f(o) = 6, f(2) = 11, f(-3) = 6, using determinants.

41) If x, y and z all positive, are p^{th}, q^{th} and r^{th} terms of a G.P. Prove that $\begin{vmatrix} \log x & p & 1 \\ \log y & q & 1 \\ \log z & r & 1 \end{vmatrix} = 0$

42) If a, b, c are in A.P. then find the value of : $\begin{vmatrix} 2y+4 & 5y+7 & 8y+a \\ 3y+5 & 6y+8 & 9y+b \\ 4y+6 & 7y+9 & 10y+c \end{vmatrix}$ (O)

43) If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then show that $A^n = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

44) If $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ find A^2 Hence find A^6 Ans: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

45) Find x, if $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$ Ans: $x = \pm 4\sqrt{3}$

46) If P and Q are invertible matrices of same order, then show that PQ is also invertible.

47) If the points (2,0), (0,5) and (x,y) are collinear give a relation between x and y.

48) Let $\begin{vmatrix} 3 & y \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$ find the possible values of x and y, find the values if x = y.

49) If $A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$ prove that $A^n = \begin{bmatrix} a^n & \frac{b(a^n-1)}{a-1} \\ 0 & 1 \end{bmatrix}, n \varepsilon N$

50) For any square matrix verify A(adj A) = |A|I

*****

RAPID FIRE ON MATRICES

1. If a matrix has 8 elements. What are the possible orders it can have? What if it has 5 elements?

2. $If \begin{bmatrix} x+3 & 2x \\ 6 & y \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 6 & 3 \end{bmatrix}, then\ find\ the\ Values\ of\ x\ and\ y.$

3. $Construct\ a\ 2 \times 2\ matrix\ A = (a_{ij}), where\ elements\ are\ given\ by\ a_{ij} = \frac{|i-2j|}{2}$

4. $Let\ A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, find\ BA.$

5. Show that the matrix $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ is skew symmetric.

6. $If\ A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}, Show\ that\ A - A^T\ is\ skew\ symmetric\ matrix.$

7. Give examples of matrices A and B such that $AB = 0\ but\ BA \neq 0.$

8. Let A be a square matrix. Then check whether $A + A^T$ is a symmetric matrix.

9. Show that the elements on the main diagonal of a skew symmetric matrix are all zero.

10. Let A and B be symmetric matrices of same order. Then show that AB - BA is a skew symmetric matrix.

11. Let A and B be symmetric matrices of same order. Then show that AB + BA is a symmetric matrix.

12. Give examples of matrices A and B such that $AB \neq BA.$

13. Show that the matrix BT A B symmetric according as A is symmetric.

14. *If* $A=\begin{bmatrix} a & 0 \\ 1 & 1 \end{bmatrix}$ *and* $B=\begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, *find the value of* α *for which* $A^2 = B$.

15. *If* $\begin{bmatrix} x & 3x-y \\ 2x+z & 3y-w \end{bmatrix}=\begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$, *find* x, y, z, w.

16. *If* $A=\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, *then find the value of* K *for which* $A^2 = KA - 2I$

ANSWERS

1. $\{1\times 8, 8\times 1, 2\times 4, 4\times 2; 5\times 1, 1\times 5\}$

2. x = 4 and y = 3

.

3. $\begin{bmatrix} -\frac{1}{2} & -\frac{3}{2} \\ 0 & -1 \end{bmatrix}$

4. $\begin{bmatrix} 11 & 10 \\ 11 & 2 \end{bmatrix}$

7. $A=\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $B=\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

14. $\alpha = \pm 1$

15. x = 3, y = 9, z = -2, w = 20

16. k = 1

RAPID FIRE ON DETERMINANTS

Q. 1.

$Evaluate: \begin{bmatrix} x & -7 \\ x & 5x+1 \end{bmatrix}.$

Q. 2.

$If\ A = \begin{bmatrix} a & c \\ b & d \end{bmatrix}, then\ find\ adjA$

Q. 3. Without expanding prove that $\begin{vmatrix} 1 & a & a^2 & -bc \\ 1 & b & b^2 & -ac \\ 1 & c & c^2 & -ab \end{vmatrix} = 0.$

Q. 4. For what Value of x the matrix $A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & x & -3 \end{bmatrix} is\ singular.$

Q. 5. If A is a matrix of order 3 and |A| = 8 , then find the value of |adj A|

Q. 6. Without expanding evaluate the determinant $\begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix}.$

Q. 7. Find the adjoint of the matrix $\begin{vmatrix} -3 & 5 \\ 2 & 4 \end{vmatrix}.$

Q. 8. $If\ A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}, then\ show\ that\ |2A| = 4|A|$

Q. 9. $Let \begin{vmatrix} 3 & y \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}.$ Find the possible values of x and y, if x, $y \in N.$

Q. 10.

$If \begin{vmatrix} 3 & m \\ 4 & 5 \end{vmatrix} = 3. Write\ the\ value\ of\ m.$

Q. 11.

$If \begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{bmatrix} 2x & 4 \\ 6 & x \end{bmatrix}$, then find the value of x.

ANSWERS

1. $5x^2 + 8x$

2. $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

4. -1

5. 64

6. 0

7. $\begin{bmatrix} 4 & -2 \\ -5 & -3 \end{bmatrix}$

8. x = 4, y = 2; x = 2, y = 4; x = 8, y = 1; x= 1, y = 8

10. m = 3

11. $x = \pm\sqrt{3}$