VECTOR ALGEBRA

PRACTICE TEST – 1

1) Find the unit vector perpendicular to both the vectors

$\bar{a} = 4\bar{i} - \bar{j} - 3\bar{k}$ and $\bar{b} = 2\bar{i} + 2\bar{j} - \bar{k}$ Ans : $\frac{1}{3}\left(-\bar{i} + 2\bar{j} + 2\bar{k}\right)$

2) If $\bar{\alpha} = 3\bar{i} - \bar{j}$ and $\bar{\beta} = 2\bar{i} + \bar{j} - 3\bar{k}$. Express $\bar{\beta}$ as a sum of two vectors $\bar{\beta}_1$ & $\bar{\beta}_2$, where $\bar{\beta}_1$ is parallel to $\bar{\alpha}$ and $\bar{\beta}_2$ is perpendicular to $\bar{\alpha}$.

3) If $\bar{a} + \bar{b} + \bar{c} = 0$, show that $\bar{a} x \bar{b} = \bar{b} x \bar{c} = \bar{c} x \bar{a}$

4) Prove the triangle inequality $|\bar{a} x + \bar{b} \leq |\bar{a}|| + |\bar{b}|$

5) Prove Cauchy - Schawarz inequality : $(\bar{a}.\bar{b})^2 \leq |\bar{a}|^2 . |\bar{b}|^2$

6) If $\bar{a}$ and $\bar{b}$ are vectors, prove that $|\bar{a} x \bar{b}|^2 + (\bar{a}.\bar{b})^2 = |\bar{a}|^2 . |\bar{b}|^2$

7) Prove that angle in a semi-circle is a right angle.

8) If $\hat{a}$ and $\hat{b}$ are unit vectors inclined at an angle θ, then prove that

a) $\cos\frac{\theta}{2} = \frac{1}{2}|\hat{a} + \hat{b}|$ b) $\tan\frac{\theta}{2} = \frac{|\hat{a} - \hat{b}|}{|\hat{a} + \hat{b}|}$

Practice Test – 2

1. Find the magnitude of two vectors $\vec{a}$ and $\vec{b}$ having the same magnitude and such that angle between them is 30° and their scalar product is 3.

2. If $\hat{a}$ and $\hat{b}$ are unit vectors and θ is the angle between them, then show that : $\sin\frac{\theta}{2} = \frac{1}{2}|\hat{a} - \hat{b}|$

Show that : $|\vec{a} \times \vec{b}| = \sqrt{a^2b^2 - (\vec{a}\,.\,\vec{b})^2}$.

3. Find the unit vector in the direction of the sum of vectors $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$

4. Show that $\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}$ is perpendicular to the plane containing the points *A*, *B*, *C* whose position vectors are $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively.

5. Show that the points $A(2\hat{i} - \hat{j} + \hat{k})$, $B(\hat{i} - 3\hat{j} - 5\hat{k})$, $C(3\hat{i} - 4\hat{j} - 4\hat{k})$ are the vertices of a right-angled triangle.

6. Find the values of *x* and *y*, so that the vectors $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ are equal.

7. Let $\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{d}\,.\,\vec{c} = 21$.

8. Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

Find $|\vec{a} - \vec{b}|$, if two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a}\,.\,\vec{b} = 4$.

ANSWERS

1. 1.86

4. $\frac{4}{\sqrt{29}}\hat{i} + \frac{3}{\sqrt{29}}\hat{j} - \frac{2}{\sqrt{29}}\hat{k}$

7. $x = 2, y = 3$

8. $7\hat{i} - 7\hat{j} - 7\hat{k}$

9. $\frac{5\sqrt{6}}{3}$

10. $\sqrt{5}$

PRACTICE TEST – 3

1. Find the values of 'p' for the vectors $3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\hat{i} + p\hat{j} + 3\hat{k}$ to be
 (*i*) perpendicular (*ii*) parallel.

2. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$.
Prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$.

3. Using vectors, show that the points $A(2, -1, 3)$, $B(4, 3, 1)$ and $C(3, 1, 2)$ are collinear.

Write the direction ratios of the vector $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$ and hence calculate its direction cosines.

4. If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, then show that the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular.

5. If $\vec{\alpha} = 3\hat{i} - \hat{j}$ and $\vec{\beta} = 2\hat{i} + 2\hat{j} - 3\hat{k}$, express $\vec{\beta}$ as a sum of two vectors $\vec{\beta_1}$, $\vec{\beta_2}$, where $\vec{\beta_1}$ is parallel to $\vec{\alpha}$ and $\vec{\beta_2}$ is perpendicular to α.

6. If $\vec{a}$ is a unit vector and $(\vec{x} + \vec{a}) \cdot (\vec{x} - \vec{a}) = 8$, then find $|\vec{x}|$.

that $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$, for any two non-zero vectors $\vec{a}$ and $\vec{b}$.

7. If $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$ and $\vec{a}$, $\vec{b}$, $\vec{c}$ are such that each is perpendicular to the sum of other two,
$|\vec{a} + \vec{b} + \vec{c}|$.

8 Find the area of a parallelogram whose adjacent sides are given by the vectors
$= 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

Answers

(i) -15 (*ii*) $\frac{2}{3}$ **4.** $1, 1, -2$; $\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}$ **6.** $\vec{\beta_1} = \frac{1}{2}(3\hat{i} - \hat{j})$, $\vec{\beta_2} = \frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$

7. **3** **9.** $5\sqrt{2}$ **10** $\sqrt{42}$ sq. units

RAPID FIRE ON VECTORS

Q. 1.
Find $\left|\vec{a}-\vec{b}\right|$, *if two Vectors* $\vec{a}$ *and* $\vec{b}$ *are such that* $\left|\vec{a}\right| = 2, \left|\vec{b}\right| = 3$ *and* $\vec{a}\cdot\vec{b} = 4.$

Q. 2. Find a unit vector perpendicular to both $\hat{i}+\hat{j}$ *and* $\hat{j}+\hat{k}$

Q. 3. For two non zero vectors $\vec{a}$ *and* $\vec{b}$ *write when* $\left|\vec{a}+\vec{b}\right| = \left|\vec{a}\right| + \left|\vec{b}\right|$ *holds?*

Q. 4.
Let $\vec{a}$ *and* $\vec{b}$ *be two Vectors such that* $\left|\vec{a}\right| = 3$ *and* $\left|\vec{b}\right| = \frac{\sqrt{2}}{3}$
and $\vec{a}\times\vec{b}$ *is a unit Vector. Then what is the angle between* $\vec{a}$ *and* $\vec{b}$

Q. 5. *Write the value of* $\hat{i}.(\hat{j}\times\hat{k})+\hat{j}.(\hat{k}\times\hat{i})+\hat{k}.(\hat{i}\times\hat{j})$

Q. 6. Find a unit vector in the direction of $\overrightarrow{a} = \hat{i}-2\hat{j}$ that has magnitude 7 units.

Q. 7. Find the direction cosines of the vector $\hat{i}+2\hat{j}+2\hat{k}.$

Q. 8. Compute the area of a parallelogram diagonal vectors are $2\hat{i}-3\hat{j}+6\hat{k}$ *and* $2\hat{i}-2\hat{j}-\hat{k}.$

Q. 9.
If θ *is the angle between two vectors* $\vec{a}$ *and* $\vec{b}$ *such that*
$\left|\vec{a}.\vec{b}\right| = \left|\vec{a}\times\vec{b}\right|$, *then find* θ

Q. 10.
$\vec{a} = 4\hat{i}+3\hat{j}+2\hat{k}$ *and* $\vec{b} = 3\hat{i}+2\hat{k}$, *find* $\left|\vec{b}\times\vec{2}a\right|.$

Q. 11. Find the angle between two vectors having same length $\sqrt{2}$ and their scalar product is -1.

Q. 12.
Find $(\vec{a}+3\vec{b}).(2\vec{a}-\vec{b})$, *if* $\vec{a} = \hat{i}+\hat{j}+2\hat{k}$ *and* $\vec{b} = 3\hat{i}+2\hat{j}-\hat{k}$

Q. 13.
For ant two vectors $\vec{a}$ *and* $\vec{b}$, *prove that* $\left|\vec{a}+\vec{b}\right| = \left|\vec{a}-\vec{b}\right| \Leftrightarrow \vec{a}\perp\vec{b}$

Q. 14.

Find $|\vec{x}|$, *if for a unit vector* $\vec{a}$, $|\vec{x} - \vec{a}| = |\vec{x} + \vec{a}| = 15$.

Q. 15.

Find the value of λ *so that Vectors* $\vec{a} = 2\hat{i}\ \lambda\hat{j} + \hat{k}$

and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ *are perpendicular to each other.*

Q. 16.

If $|\vec{a} \times \vec{b}| = 4, \quad |\vec{a} \cdot \vec{b}| = 2$, *then find* $|\vec{a}|^2\ |\vec{b}|^2$.

Q. 17.

If $\vec{a}\,.\hat{i} = \vec{a}\,.\,(\hat{i} + \hat{j}) = \vec{a}\,.\,(\hat{i} + \hat{j} + \hat{k}) = 1$, *then find* $\vec{a}$.

Q. 18. Find the projection of the vector $7\hat{i} + \hat{j} + 4\hat{k}$ *on* $2\hat{i} + 6\hat{j} + 3\hat{k}$.

Q. 19.

Find $|\vec{a}|$ *and* $|\vec{b}|$. *If* $(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 8$ *and* $|\vec{a}| = 8\,|\vec{b}|$

ANSWERS

1. $\sqrt{5}$
2. $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$
3. *When* $\vec{a}$ *and* $\vec{b}$ *are collinear*
4. $\frac{\pi}{4}$
5. 3
6. $\frac{7}{\sqrt{5}}(\hat{i} - 2\hat{j})$

7. $\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$

8. $\frac{5\sqrt{17}}{2}$ $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

9. $\frac{\pi}{4}$

10. $\sqrt{404}$

11. $\frac{2\pi}{3}$

12. - 15

14. $|\vec{x}| = 4$

15. $\frac{5}{2}$

16. 20

17. $\hat{i}$

18. $\frac{8}{7}$

19. $|\vec{b}| = \sqrt{\frac{8}{63}}, |\vec{a}| = \frac{8\sqrt{8}}{\sqrt{63}}$